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Modeling and Control of a 3 DOF Monoped Robot

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Abstract

Interests and motivations towards enhanced humanoid robots considering their applications have grown in recent years. In the present stuudy a 3–link monoped robot is studied. This robot can be used in several applications such as artificial legs and biped robots. We have simulated and controlled a model of the monoped robot by "the inverse dynamics" method and concluded that an adaptive scheme is required to set the proportional and derivative gains during different giat phases.

Keywords: Walking Model – Gait – Inverse Dynamics Control.

1. Introduction

Study of human walking is a very familiar subject in Biomechanical Engineering. Many studies have been conducted to model the walking gait of human, also running, jumping or many other gaits. The purpose of almost all these studies has been finding the forces and moments that act in joints or on body parts, which can be used in designing artificial parts or joints.

On the other hand, motions similar to human walking are needed in some robotic applications such as house keeping services, rescuing maneuvers in high risk situations for human beings, artificial legs improvement, etc. However in order to simulate the human walking using current robotics knowledge there are some difficulties which should be reviewed.

In this paper, a 3–link model for a walking robot is considered. This model is controlled by the inverse dynamics method. The model is expected to simulate a typical human walking of a real foot. The experimental data are taken from reference [1]. These data are discrete and are collected in 69.9 frames per second resulting in a precision of at most 1/69.9=0.0143 sec.

In section 2 the problems encountered when trying to reach a walking model and the model used in this study are presented while the control strategy is discussed in section 3. Finally the simulation results and conclusions are given in section 4.

2. Modeling of the Human Walking

Modeling a mobile robot generally has more difficulties than a fixed robot. Several approaches have been investigated for mobility of the robot base. For instance, one can use the modified formulation of fixed robots for mobile robots which is however relatively complex. Another popular solution in walking studies is to assume the mobile robot as a fixed robot on a moving ground. In the case of low acceleration motions like slow walking we can suppose the hipbone as the base hinge.

The next crucial point which should be regarded in walking simulation is the number of degrees of freedom (DOF) we would like to consider in the model. As a complex natural behavior, the human walking process has a large number of DOF. Walking is a rhythm and each of its movements is forecasted and certain, but is strongly controlled by

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a hierarchical control system, which works from muscles to the brain and spanned from cells to the whole body.

The body members that contribute in walking are from a linkage called Lower Limb. The lower limb linkage consists of five parts: from base rib cage to the joint of thigh is named Hip. From the joint of thigh to knee is Thigh. From knee to malleolus is Leg or Shank. From ankle to the joint of foot fingers is Metatarsal or Foot, and from the joint to the tip of toe is Toe. Here metatarsal is the name of joint of foot fingers. In walking kinematics, all foot fingers are taken as one and called Toe. The two important points in walking process are toe tip (briefly called toe) and heel, because in walking, metatarsal and fingers touch the ground, push the body forward, support body stability during walking, etc. They are the last two members of lower limb linkage and their positions/orientations are the results of operation of lower limb.

Walking is divided into two phases: swing and stance. In swing phase, leg is suspended and moves forward to reach new positions. During stance, leg stands on the ground to support body stability. Before stance phase starts, foot turns around ankle to cause heel be the lowest point of lower limb and touches the ground at first. At this moment lower limb slows down to prevent impact of heel with the ground. After that heel touches the ground, foot fingers lay on the ground to provide stability support; body moves forward and leg turns backward (clockwise). After the moment that body and leg are in a vertical line, foot comes apart from the ground and fingers stay in touch with the ground. From this moment till fingers take off from the ground, their duty is to push body forward and produce the thrust force for walking. At the time that fingers take off from the ground, swing phase starts, Knee bends and fingers turn upward to prevent foot from contacting with the ground. When knee passes the bodyline, it is time for knee to come open and shank be triggered forward to reach new position. Then the metatarsal of other foot comes apart from the ground. By opening the knee angle, foot turns upward to cause that heel be the first point that touches the ground. The body trunk and head stay vertical during all this process. The major motion of body is in forward direction and almost in steady speed; but it oscillates some centimeters in the vertical direction.

Of course modeling of such a complex system demands for a large number of DOFs. In the present work however we would like our robot to approach some features of the real walking behavior. In other words reduced models when insightfully designed may capture some domains of the complex model behavior. This is what we have learned from some revolutionary theories such as the chaos theory [2].

Therefore a 3–link mechanism as shown in Fig.1 would be the first natural choice. We use the real data of thigh and leg angles shown in Fig.2 which are due to a 31.4 cm. thigh length and 42.5 cm. leg length. The foot angle is desired to be always 180 degrees or parallel to the ground with the toe tip directed forward. To be more consistent with the natural walking, a sinusoidal vertical trunk motion with amplitude of 2.5 cm. is also required.

The contact force on the foot link (link number 3) is modeled as:

$$f\_ext = [V^T, M^T] = [V_x, V_y, V_z, M_x, M_y, M_z]$$

where V is the external force and M is the external moment applied on the end point (toe tip) of the final link and stated in the link coordinate frame. In fact we have assumed that the distributed external load on the foot sole can be replaced equivalently (for static analysis) by a concentrated force on the middle of the sole which can further more be replaced by an equivalent force and a moment on the toe tip(Fig. 2).

Some robot specifications required to solve kinematics and dynamics problems such as links lengths, links masses, inertia matrices and location of the center of masses are presented in table 1. The Denavit-Hartenberg table can be obtained knowing the links joints and orientations from Fig.1 [3].

3. Control Strategy

Typically, Control design for rigid robot manipulators is based on the Lagrange dynamics equations of motion:

$$H(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) = u \tag{1}$$

where q is the $n \times 1$ vector of generalized join coordinates, $H(q)$ is the $n \times n$ inertia matrix, $g(q)$ is the $n \times 1$ vector of gravity force, $C(q,\dot{q})\dot{q}$ is the $n \times 1$ vector of Coriolis and centrifugal forces and u is the joint control input torques to be designed.

Mechanical intuition dictates the idea of canceling nonlinear terms and decoupling the dynamics of each link by using an "*inverse dynamics*" control of the form:

$$u = H(q)u_0 + C(q,\dot{q})\dot{q} + g(q) \tag{2}$$

Typical choices for u_0 are:

$$u_0 = \ddot{q}_d + K_d(\dot{q}_d - \dot{q}) + K_p(q_d - q) \tag{3}$$

or with an integral component:

$$u_0 = \ddot{q}_d + K_d(\dot{q}_d - \dot{q}) + K_p(q_d - q) + K_i\int_0^t (q_d - q)d\tau \tag{4}$$

The index d refers to the desired joint coordinates. Also the K_d and K_p are the derivative and proportional positive definite gain matrices. We use (3) in this work which leads to the error equation:

$$(\ddot{q}_d - \ddot{q}) + K_d(\dot{q}_d - \dot{q}) + K_p(q_d - q) = 0 \tag{5}$$

The error equation is exponentially stable by a suitable choice of the matrices K_p and K_d [4]. For the forward and inverse dynamics parts of this control scheme (equations (1) and (2)) we use Newton – Euler formulation. This formulation allows computing the dynamic model of a rigid manipulator without deriving the explicit expression of the terms H , C and g . The equations of motion are obtained as the result of two recursive computations; namely, a forward recursion to compute link velocities and accelerations from link 1 to link 3, and a backward recursion to compute link forces and moments (and thus the joint torques) from link 3 to link 1. The details of this procedure are presented in several robotics texts [3, 4].

The construction of the model, forward kinematics, forward and inverse dynamics are implemented using the Robotics Toolbox for MATLAB [5].

4. Simulation Results and Conclusion

Several gait phases may occur during a real behavior from slow walking to running. One way to distinguish these different phases is the step duration. We have changed this parameter from 1 seconds to 3 and presented the results. In order to provide an overview of what happens for model under control, the path of the heel (the end point of link number 2) and the foot angle are presented in Fig.4 to Fig. 8. The desired paths represented as dashed lines in these figures, resemble an inclined crescent or banana. This is predictable. In fact the almost horizontal (flat) segment of this crescent corresponds to the stance phase and the other parts are due to the swing phase as described in section 2. As the dominant case, we first consider the step duration equal to 2 seconds and next the cases due to step durations of 1 and 3 seconds will be discussed respectively. Fig. 4 shows the foot desired and controlled angles when the step duration is 2 seconds, $K_p = 5I_{3\times3}$ and $K_d = 23I_{3\times3}$. This figure shows a small error of about -0.11 degrees in foot angle in the end of the step. Fig. 5 shows the heel desired and control paths for this case. As we see there is a deviation of about 5 cm. from the desired path in regions where there is a rapid change in slope; so we expect the derivative gain K_d be responsible for this deviation.

Shown in Fig.6 is the foot desired and controlled angles for the step duration equal to 1 second, $K_p = 5I_{3\times3}$ and $K_d = 23I_{3\times3}$ while the heel paths are presented in Fig.7. In this case the deviation from the desired path is about 25 cm. This result is predictable since the step duration has reduced in this case and consequently the robot experiences a faster change of slope than that in the previous case. So increasing the derivative gain as shown in Fig.8 and Fig.9 must lead to better results. Fig.10 and Fig.11 demonstrate the results for step duration equal to 3 seconds with $K_p = 5I_{3\times3}$ and $K_d = 23I_{3\times3}$ as in the first case. Here the improperness of these gains is more serious than the previous cases as Fig.11 shows an unstable behavior. It seems that the control is too sensitive in this case. So a reduction in K_d is expected to be the solution. The result due to $K_d = 15$ shown in Fig.13 justifies this reasoning.

In conclusion it can be said that the inverse dynamics control combining with a gain scheduling procedure for K_d produces acceptable results.

References

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Table 1 - The robot specifications

Link	Length(cm)	Center of Mass distance to the first joint (cm)
Thigh	31.4	15.7
Leg (Shank)	42.5	21.25
Foot	20	10

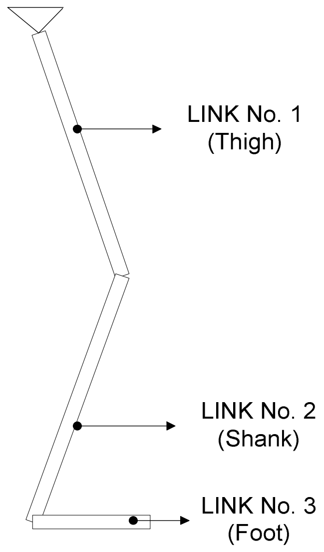

Fig. 1: The Thigh – shank – foot linkage model

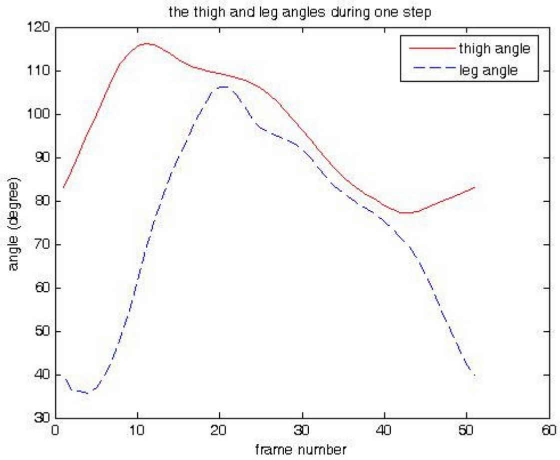

Fig2: The thigh and leg angles (relative to the ground in counterclockwise direction) during a real gait due to a 31.4 cm. thigh length and 42.5 cm. leg length.

Table 1 – continued

Link	Mass (kg)
Thigh	6
Leg (Shank)	5
Foot	1

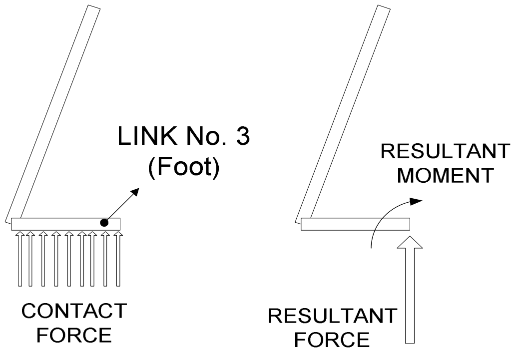

Fig. 3: The distributed contact force on the foot sole can be equivalently (for static analysis) replaced by a concentrated force and moment on the tip.

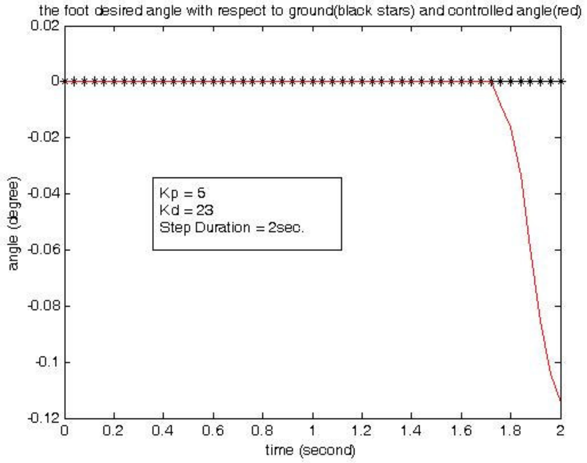

Fig.4: The foot desired and controlled angles for step duration = 2 sec., $K_p = 5I_{3\times3}$ and $K_d = 23I_{3\times3}$

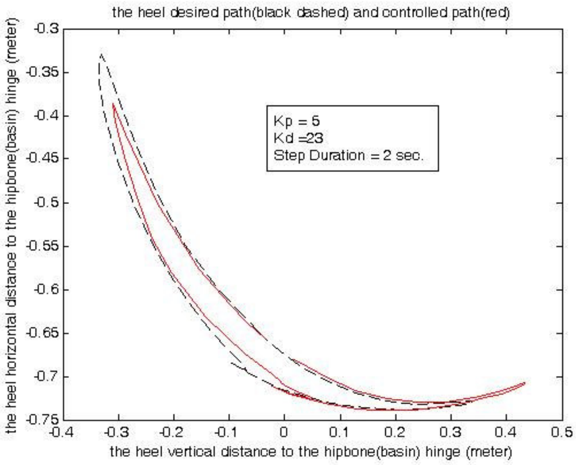

Fig.5: The heel desired and controlled paths for step duration = 2 sec., $K_p = 5I_{3\times3}$ and $K_d = 23I_{3\times3}$

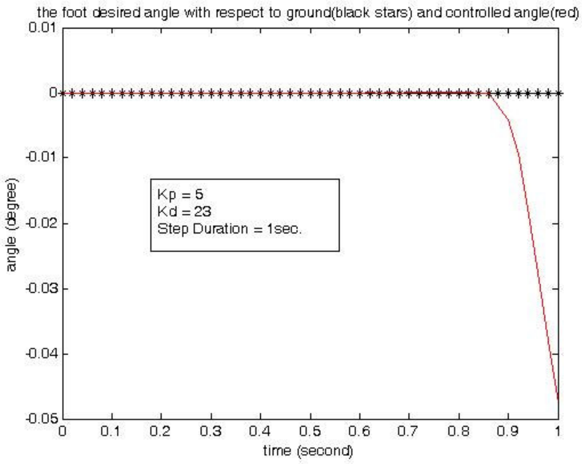

Fig.6: The foot desired and controlled angles for step duration = 1 sec., $K_p = 5I_{3\times3}$ and $K_d = 23I_{3\times3}$

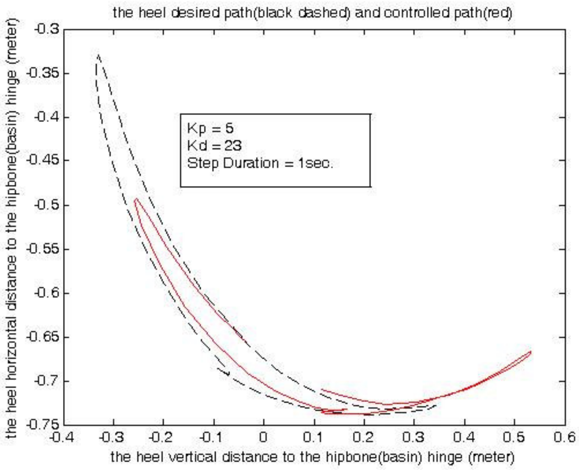

Fig.7: the heel desired and controlled paths for step duration = 1 sec., $K_p = 5I_{3\times3}$ and $K_d = 23I_{3\times3}$

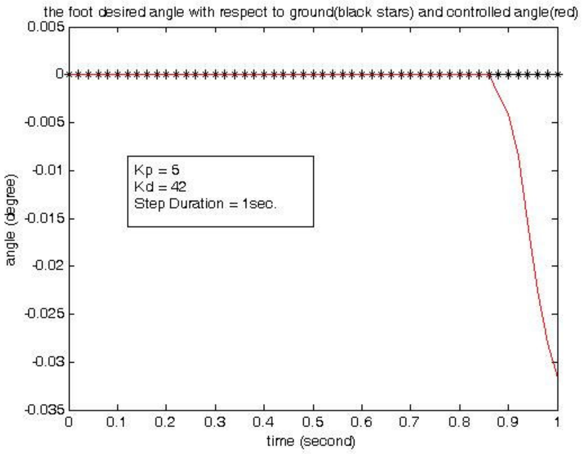

Fig.8: The foot desired and controlled angles for step duration = 1 sec., $K_p = 5I_{3\times3}$ and $K_d = 42I_{3\times3}$

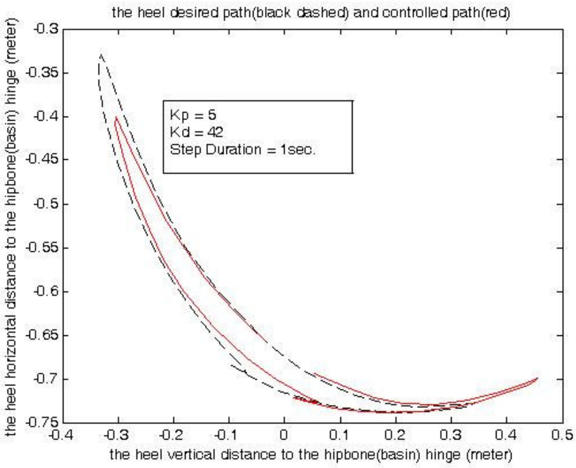

Fig.9: The heel desired and controlled paths for step duration = 1sec., $K_p = 5I_{3\times3}$ **and** $K_d = 42I_{3\times3}$

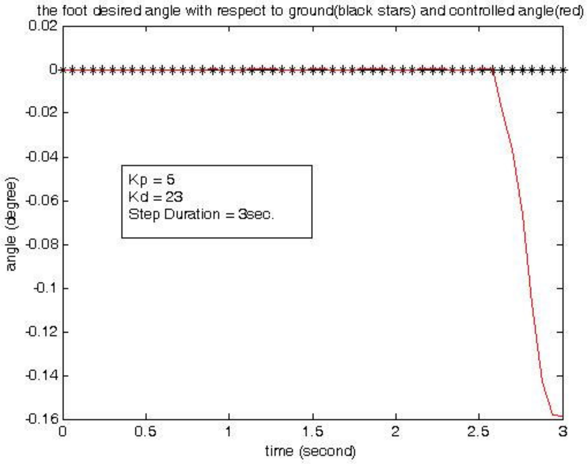

Fig.10: The foot desired and controlled angles for step duration = 3sec., $K_p = 5I_{3\times3}$ **and** $K_d = 23I_{3\times3}$

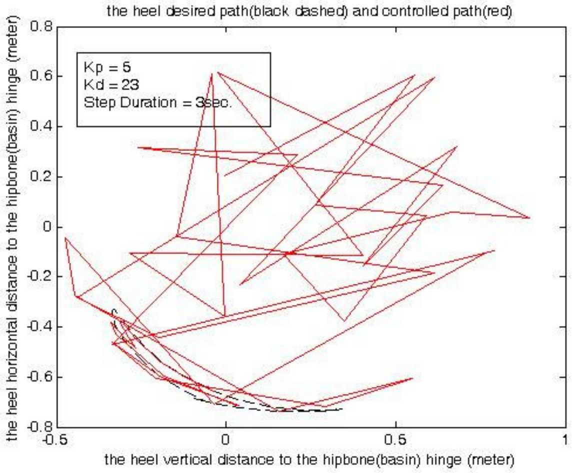

Fig.11: The heel desired and controlled paths for step duration = 3sec., $K_p = 5I_{3\times3}$ **and** $K_d = 23I_{3\times3}$

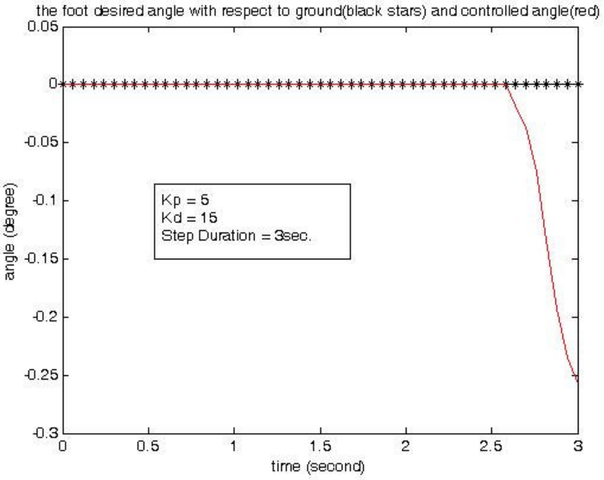

Fig.12: The foot desired and controlled angles for step duration = 3sec., $K_p = 5I_{3\times3}$ **and** $K_d = 15I_{3\times3}$

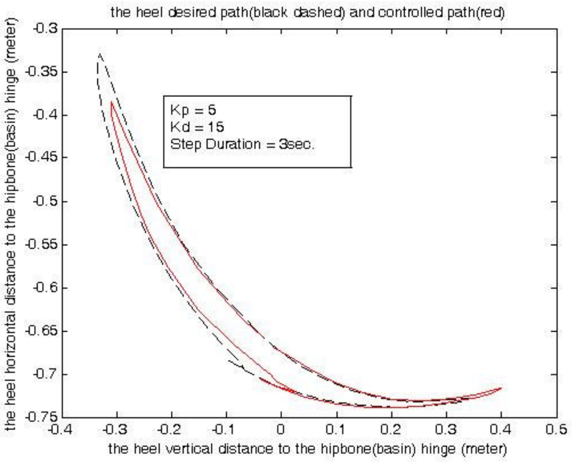

Fig.13: The heel desired and controlled paths for step duration = 3sec., $K_p = 5I_{3\times3}$ and $K_d = 15I_{3\times3}$